Diversions

with Barry Squire

Paul Scott's Diversions are a hard act to follow — some great attention grabbers and snappy classroom activities — well done Paul! Rather than try to continue in the same vein, I am happier to present over the next few issues some investigations into aspects of number patterns that have engaged students of mathematics for many years and been a subject of interest for me since I was a schoolboy myself. It is left for you as the teacher to select and fashion any aspects you feel suit your own students. So, here we go!

Classy numbers in good shape! or Roamin' in the gnomon

No, you are not about to see a healthy looking centrefold! Rather this is to be an excursion into an all-encompassing subset of our whole numbers, or integers, which of course are the building blocks of the branch of mathematics called Number Theory.

The integer 1 belongs to all of the sets and we know it is classed as 'odd' along with 3, 5, 7, etc. as distinct from 'even' numbers, which are all the others 2, 4, 6, etc. By the way, why call them 'odd' and 'even'?

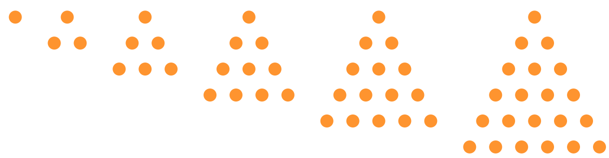
Another familiar set of numbers is the set of 'square numbers'. Why is that familiar? Perhaps because it is a familiar shape in our environment and the number can be represented as a square array of dots, in a grid with equal numbers of rows and columns, such as 4 = 2 rows of 2; 9 = 3 rows of 3, etc. I suppose that's as far as most people go in their investigation of numbers linked with familiar geometric shapes, but why stop there? Might there be other shapes to numbers such as triangles, pentagons, hexagons, etc? Is there any limit?

As long ago as Pythagoras (well known for 'his' theorem about the right angled triangle, but he wrote down much about arithmetic and number theory as well) this progression of geometric shape names was linked to sequences of numbers that follow rather nice patterns that we now explore.

Figurate numbers

A figurate number is a number that can be represented as a regular and discrete geometric pattern (e.g. dots).

The first few triangular numbers can be built from rows of 1, 2, 3, 4, 5, and 6 items. (A neater way to show them is using some isometric 'dotty' paper and a highliter pen to mark out successive groups of the dots with 1, 2, 3, 4, 5 and 6 dots along each 'side' of the triangular arrays.) For now we have:



So, the total number of dots in each column here are the successive triangular numbers. What are they? Have you seen them happening in other contexts? And the more interesting question is to see if you can find the general form for these numbers! (We might well get to that in the next issue.)

A trick that is useful to explore is to find the pattern of the numbers that you add to get the next number in the sequence. This is called the *gnomon*. Strange word? Its definition is

linked in history to our investigation of figurate numbers. Here is a bit of information on this special word and a neat exercise involving our earlier set of numbers, the odd numbers, that we will apply later to these triangular numbers and see what happens.

Gnomon

Figurate numbers were a concern of Pythagorean geometry, (I just love these links between numbers and shapes!) since Pythagoras is credited with initiating them, and the notion that these numbers are generated from a gnomon or basic unit.

The gnomon is the piece which needs to be added to a figurate number to transform it to the next biggest one.

As an example, let's look at the square numbers and see what these gnomons look like as we progress through squares of sides 1, 2, 3, 4, 5, 6, 7 and 8 as illustrated with a grid of numbers that really gives us the answers quite readily:

8	8	8	8	8	8	8	8
8	7	7	7	7	7	7	7
8	7	6	6	6	6	6	6
8	7	6	5	5	5	5	5
8	7	6	5	4	4	4	4
8	7	6	5	4	3	3	3
8	7	6	5	4	3	2	2
8	7	6	5	4	3	2	1

Write down the successive gnomons that add to form the next square. (They are shown as the number of 2s, 3s, 4s, etc.)

You should recognise this sequence of numbers! What is the *general term* for them?

Back to our earlier triangular numbers. Tables are good problem solving tools so let's enter our findings in a table and see where that leads us in finding a general term for these critters!

Fill in a table like the one here for the triangular numbers (I have filled in a few to start you off — go further to 7 and 8):

Dots in row	1	2	3	4	5	6	7	8
Total dots in number	1	3	6					
Gnomon	2	3						

So how do you get from one triangular number to the next one? What is the sequence of gnomons this time? Any surprise?

Summing up, for the triangular numbers, the gnomons were the set of ________________ numbers; for the square numbers the gnomons were the set of ________________ numbers. What might the gnomons for the next shaped figurate numbers be? And the next? And the next?

Actually, a famous mathematician, often called the *Prince of Amateurs*, Pierre Fermat (1601–1665) declared that any number is either a *figurate* number or the *sum of two or more of them*. His actual statement is a lot more detailed than that and might be of interest; consult the Web via your favourite browser. It is an amazing statement and typical of Fermat's genius and insight! You might like to test his statement for the numbers 1 to 20 or so.

Quick adder

Find the quickest way to add the numbers from 1 to 100. If you can do it in 2 steps you might be as smart as a famous mathematician! Who was he and what is the story told about him as a boy?
(We'll use his technique to revisit figurate numbers next issue! See you then!)

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